# UNIVERSITY DEPARTMENT OF MATHEMATICS

## Tilka Manjhi Bhagalpur University, Bhagalpur

**Assignment – I**

Due Date: 28–09-19 | PAPER – XV | Session: 2018–20

1. Solve the following problems using simplex method

   (a) Solve the linear program

   $$\begin{aligned} \text{minimize: } & x_1 - 2x_2 - 4x_3 + 2x_4 \\ \text{subject to: } & x_1 - 2x_3 \le 4 \\ & x_2 - x_4 \le 8 \\ & -2x_1 + x_2 + 8x_3 + x_4 \le 12 \\ & x_1, x_2, x_3, x_4 \ge 0 \end{aligned}$$

   (b) Solve the linear program

   $$\begin{aligned} \text{minimize: } & 2x - y + 2z \\ \text{subject to: } & 2x + y \le 10 \\ & x + 2y - 2z \le 20 \\ & y + 2z \le 5 \\ & x, y, z \ge 0 \end{aligned}$$

   Answer: Optimal is 15 at $(5, 0, 5/2)$.

   (c) Solve the linear program

   $$\begin{aligned} \text{maximize: } & x_1 + 2x_2 + 2x_3, \\ \text{subject to: } & 5x_1 + 2x_2 + 3x_3 \le 15 \\ & x_1 + 4x_2 + 2x_3 \le 12 \\ & 2x_1 + x_3 \le 8 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

   (d) Solve the linear program

   $$\begin{aligned} \text{maximize: } & 4x_1 + 3x_2 + 6x_3, \\ \text{subject to: } & 3x_1 + x_2 + 3x_3 \le 30 \\ & 2x_1 + 2x_2 + 3x3 \le 40 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

2. Solve the following problems using Big-M and Two-phase method

   (a) Solve the linear program

   $$\begin{aligned} \text{maximize: } & 3x_1 - x_2 \\ \text{subject to: } & 2x_1 + x_2 \ge 2 \\ & x_1 + 3x_2 \le 2 \\ & x_2 \le 4 \\ & x_1, x_2 \ge 0 \end{aligned}$$

   (b) Solve the linear program

   $$\begin{aligned} \text{maximize: } & 2x_1 + 3x_2 + 4x_3 \\ \text{subject to: } & 3x_1 + 2x_2 + x_3 \le 10 \\ & 2x_1 + 3x_2 + 3x_3 \le 15 \\ & x_1 + x_2 - x_3 \ge 4 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

   Answer: Optimal is 140/9 at $(1/3, 38/9, 5/9)$.

3. Infeasibility problems

   (a) Show that the following problem has no feasible solution

   $$\begin{aligned} \text{maximize: } & 2x + 5y \\ \text{subject to: } & 3x + 2y \ge 12 \\ & 2x + y \le 4 \\ & x, y \ge 0 \end{aligned}$$

   (b) Consider the following problem, in the phase I, the artificial variable didn't leave but assumes the value 0, hence we can remove it and continue with phase II

   $$\begin{aligned} \text{maximize: } & 2x_1 + 2x_2 + 4x_3 \\ \text{subject to: } & 2x_1 + x_2 + x_3 \le 2 \\ & 3x_1 + 4x_2 + 2x_3 \ge 8 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

4. Consider the following linear programming problem

$$\begin{aligned} \text{maximize: } & 3x + 2y \\ \text{subject to: } & 4x - y \leq 4 \\ & 4x + 3y \leq 6 \\ & 4x + y \leq 4 \\ & x, y \geq 0 \end{aligned}$$

   (a) Show that the problem is degenerate but didn't cycle.

   (b) Verify the result by solving the problem graphically.

5. Problems on alternative optima

   (a) Show that the following problem has alternative optima, hence find all the solutions

$$\begin{aligned} \text{maximize: } & 2x + 4y \\ \text{subject to: } & x + 2y \leq 5 \\ & x + y \leq 4 \\ & x, y \geq 0 \end{aligned}$$

   (b) Identify three alternative optimal basic solution, hence write the a general expression for the solutions of the following LPP.

$$\begin{aligned} \text{maximize: } & x_1 + 2x_2 + 3x_3 \\ \text{subject to: } & x_1 + 2x_2 + 3x_3 \leq 10 \\ & x_1 + x_2 \leq 5 \\ & x_1 \leq 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

6. Problems on unbounded objective

   (a) Show that the following problem has unbounded objective

$$\begin{aligned} \text{maximize: } & 2x + y \\ \text{subject to: } & x - y \leq 10 \\ & 2y \leq 40 \\ & x, y \geq 0 \end{aligned}$$

   (b) Show that the following problem has unbounded objective

$$\begin{aligned} \text{maximize: } & 20x_1 + 5x_2 + x_3 \\ \text{subject to: } & 3x_1 + 5x_2 - 5x_3 \leq 50 \\ & x_1 + 3x_2 - 4x_3 \leq 20 \\ & x_1 \leq 10 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$