PAPER – III | ASSIGNMENT – II | Linear Algebra

1. Problems on invariant subspace
   (a) For each of the following linear operator T on V and subspace W, determine if the given subspace is invariant or not
      i. $V = \mathcal{P}_3$, $T(f)(x) = f'(x)$ and $W = \mathcal{P}_2(x)$
      ii. $V = \mathbb{R}^3$, $T(a,b,c) = (a+b+c, a+b+c, a+b+c)$ and $W = \{(t,t,t) : t \in \mathbb{R}\}$
      iii. $V = M_{2\times 2}(\mathbb{R})$, $T(A) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} A$ and $W = \{A \in V : A^T = A\}$
   (b) Let T be any linear transformation from V to V, then show that the following spaces are invariant under T
      i. $\{0\}$ and V
      ii. Nullspace of T and Range of T
      iii. Space generated by any non-empty set of eigenvectors of T
      iv. Generalized eigenspace E_λ, for some eigenvalue λ

2. Problem on Jordan Canonical Forms
   (a) Find the characteristic polynomial, minimal polynomial, and the Jordan canonical form of the following matrices
      i. $\begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$
      ii. $\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$
      iii. $\begin{pmatrix} 11 & -4 & -5 \\ 21 & -8 & -11 \\ 3 & -1 & 0 \end{pmatrix}$
      iv. $\begin{pmatrix} 4 & 1 & 0 \\ -1 & 2 & 0 \\ 1 & 1 & 3 \end{pmatrix}$
      v. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 1 & 0 & 2 \end{pmatrix}$.
      vi. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 1 & -1 & 3 \end{pmatrix}$
   (b) Let $A = \begin{pmatrix} 5 & -1 \\ 9 & -1 \end{pmatrix}$. Then find the formula for A^n, where n is a positive integer.