UNIVERSITY DEPARTMENT OF MATHEMATICS
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PAPER – III | ASSIGNMENT – I | Linear Algebra

1. Problems on system of linear equation
   (a) Solve the following system of linear equations using the Gaussian elimination method and describe the type of system of linear equations using solutions

   i.
   $$\begin{aligned} 2x + y + 3z &= 1 \\ 2x + 6y + 8z &= 3 \\ 6x + 8y + 18z &= 5 \end{aligned}$$

   ii.
   $$\begin{aligned} x + 2y - 3z + w &= -2 \\ 3x - y - 2z - 4w &= 1 \\ 2x + 3y - 5z + w &= -3 \end{aligned}$$

   iii.
   $$\begin{aligned} x + y + 2z &= -2 \\ 3x - y + 14z &= 6 \\ x + 2y &= -5 \end{aligned}$$

   iv.
   $$\begin{aligned} 2x + 3y - 5z + w &= -3 \\ 3x - y - 2z - 4w &= 1 \end{aligned}$$

   (b) For what value of k does the following system of linear equation has infinitely many solutions
   $$\begin{aligned} x + ky + z &= 1 \\ y + z &= 2 \\ x + y + z &= 3 \end{aligned}$$

2. Problem on finding coordinate in given vector space
   (a) Find the coordinate of $x = (8, -9, 6)$ w.r.t the basis $\beta = \{(1, -1, 3), (-3, 4, 9), (2, -2, 4)\}$ of $\mathbb{R}^3$.
   (b) Let $\mathcal{P}_3$ be the set of all polynomial of degree less than or equal to 3. Find the coordinate of the vector $f(x) = 1 + x + x^2 + x^3$ and $g(x) = -3 + 2x^3$ relative to the following basis of $\mathcal{P}_3$
   $$\beta = \{1 - x, 1 + x, x^2 - x^3, x^2 + x^3\}$$
   (c) Consider the the vector space of all 2×2 matrices of real numbers with basis
   $$\beta = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right\}.$$
   Find the coordinate of $A = \begin{pmatrix} 5 & 4 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & 6 \\ 2 & 2 \end{pmatrix}$, with relative to β.

3. Problem on matrix of linar transformation
   (a) Let T be the linear map from $\mathbb{R}^3 \to \mathbb{R}^3$ is defined by
   $$T(x, y, z) = (3x + 2y + z, x + 3z, y + 4z),$$
   then find the matrix of linear transformation given by standard basis.
   (b) Find the matrix of linear transformation that reflects any vector along the the line inclined at an angle θ with $x-$axis.
   (c) Find the matrix of linear transformation that rotates any vector at an angle θ.
   (d) For an integer $n > 0$, let $\mathcal{P}_n$ denote the vector space of polynomials with real coefficients of degree n or less. Define the map $T : \mathcal{P}_2 \to \mathcal{P}_4$ by
   $$T(f)(x) = f(x^2).$$

Question 3 continues on the next page. . .

i. Determine if T is a linear transformation.

ii. If it is, find the matrix representation for T relative to the basis $B = \{1, x, x^2\}$ of $\mathcal{P}_2$ and $C = \{1, x, x^2, x^3, x^4\}$ of $\mathcal{P}_4$.

(e) Let $\mathcal{P}_1$ be the vector space of all real polynomials of degree 1 or less. Consider the linear transformation $T : \mathcal{P}_1 \to \mathcal{P}_1$ defined by

$$T(ax + b) = (3a + b)x + a + b,$$

i. With respect to the basis $B = 1, x$, find the matrix of the linear transformation T.

ii. Find a basis B of the vector space $\mathcal{P}_1$ such that the matrix of T with respect to B is a diagonal matrix.

iii. Express $f(x) = 5x + 3$ as a linear combination of basis vectors of B.