UNIVERSITY DEPARTMENT OF MATHEMATICS

Tilka Manjhi Bhagalpur University, Bhagalpur

Class Test Examination – 2018

Time: $1\frac{1}{2}$ hour | PAPER – III | Session: 2018–20

1. Choose the most appropriate option. (05)

   (a) Let A be a real square matrix of order n over with all it's eigenvalues belong to $\mathbb{R}$. Then which of the following statement is true.

   A. A is diagonalizable.
   B. A is symmetric.
   C. A has orthogonal eigenvecotrs.
   D. None of these

   (b) Let A be a square matrix with real entries, then which of the following is not true

   A. A is dianolizable over $\mathbb{C}$
   B. A is triangulizable over $\mathbb{C}$
   C. A is triangulizable over $\mathbb{R}$
   D. None of these

   (c) Let x_1 and x_2 be the solution of a system of linear equation $Ax = b$, and α and β be any number, then the linear combination $\alpha x_1 + \beta x_2$ will be a solution of $Ax = 0$ for

   A. any real value of α and β
   B. for infinitely many value of α and β
   C. for unique value of α and β
   D. None of these

   (d) A system of linear equation with m equations and n variables has unique solution if

   A. $\operatorname{rank} A = \operatorname{rank} \tilde{A}$
   B. $\operatorname{rank} A \neq \operatorname{rank} \tilde{A}$
   C. $\operatorname{rank} A = \operatorname{rank} \tilde{A} = m$
   D. None of these

   (e) Let $T : V \to V$ be a linear operator with eigenvalue λ, then which of the following subspace is not an invariant subspace

   A. Eigenspace corresponding to λ
   B. Generalized eigenspace corresponding to λ
   C. Range space of T, i.e., $T(V)$
   D. None of these

2. Answer any two of the following (10)

   (a) Solve the following system of linear equation using Gaussian elimination method and describe the type of system of linear equations using solutions

   $$\begin{aligned} x + 2y + 3z &= 1 \\ -3x - 2y - z &= 2 \\ 4x + 4y + 4z &= 3 \end{aligned}$$

   (b) Define the following terms with examples

   i. Generalized eigenvector
   ii. Matrix of linear transformation

   (c) Find the characteristic polynomial, minimal polynomial, and Jordan canonical form of the following matrix

   $$\begin{pmatrix} 4 & 1 & 0 \\ -1 & 2 & 0 \\ 1 & 1 & 3 \end{pmatrix}$$