1. Problems on singulariy of complex functions
   (a) Find the singularity and it's type of the following functions

      i. $f(z) = \dfrac{z^2}{1-\cos z}$ ii. $f(z) = \dfrac{z^2}{\sin(z)}$ iii. $f(z) = \dfrac{1}{e^z - 1}$

      iv. $f(z) = \dfrac{\sin z^2}{z^2(z-2)}$ v. $f(z) = \sin\left(\dfrac{1}{z}\right)$ vi. $f(z) = \dfrac{1}{1-\cos^2 z}$

   (b) Show that the singularity of $\sin\left(\dfrac{1}{z}\right)$ and $e^{1/z}$ is isolated and essential.

   (c) Show that the singularity of $f(z) = \dfrac{1}{1-\cos\left(\frac{1}{z}\right)}$ and $f(z) = \dfrac{1}{e^{\frac{1}{z}} - 1}$ at $z = 0$ is non-isolated and essential.

2. Find the radius convergence of following power series

   i. $\sum \dfrac{n!}{n^n} z^n$ ii. $\sum n^n z^n$ iii. $\sum \dfrac{(n!)^3}{3n!} z^n$

   iv. $\sum (\log n)^2 z^n$ v. $\sum [2 + (-1)^n]^n z^n$ vi. $\sum z^{n!}$

3. Question on Taylor and Laurent series
   (a) Find the Maclaurin series expansion of the function

      i. $f(z) = \dfrac{z}{z^4 + 9}$ ii. $f(z) = z^2 e^{3z}$ iii. $f(z) = \sin z^2$

   (b) Find Taylor's series expansion of $\dfrac{1}{1-z}$ around the point i.

   (c) Find the Laurent series for the function $\dfrac{z}{(z+1)(z-2)}$ in each of the following domains.

      i. $|z| < 1$ ii. $1 < |z| < 2$ iii. $|z| > 2$

   (d) Find the Laurent series expansion of $\dfrac{(z+1)}{z(z-4)^3}$ in the domain $0 < |z-4| < 4$.

   (e) Find the Laurent series for the function $z^2 \cos\left(\dfrac{1}{3z}\right)$ in the domain $|z| > 0$.

4. Find the zeros and poles of the following functions with their order

   i. $f(z) = \tan z$ ii. $f(z) = \dfrac{1}{z(e^z - 1)}$

   iii. $f(z) = \dfrac{z+3}{z^2(z^2+4)}$ iv. $f(z) = \dfrac{z-1-i}{z^2 - (4+3i)z + (1+5i)}$

5. Problem of Argument Principle
   (a) Solve the integration $\oint_{|z|=1} \dfrac{f'}{f} dz$, where f is the following

 Question 5 continues on the next page...

i. $f(z) = z^2$ ii. $f(z) = \dfrac{z^3+2}{z}$ iii. $f(z) = \dfrac{(2z-1)^7}{z^3}$

(b) Solve the integration $\oint_{|z|=10} f(z)dz$, where f is the following

i. $f(z) = \cot z$ ii. $f(z) = \dfrac{e^z}{1+e^z}$ iii. $f(z) = \dfrac{\sin z}{1-\cos z}$

6. Problems on Rouches Theorem
   (a) Determine the roots of $z^7 - 4z^3 + z - 1 = 0$ inside the circle $|z| = 1$.
   (b) Determine the number of zeros, counting multiplicities, of the polynomial $z^4 - 2z^3 + 9z^2 + z - 1 = 0$ inside the circle $|z| = 2$.
   (c) If $a > e$, then show that the equation $az^n = e^z$ has n roots inside $|z| = 1$.
7. Problems on Residue Calculus
   (a) Find the residue at $z = 0$ of the function

   i. $\dfrac{1}{z+z^2}$ iii. $\dfrac{z-\sin z}{z}$

   ii. $z\cos\left(\dfrac{1}{z}\right)$ iv. $\dfrac{\cot z}{z^4}$

   (b) Use Cauchy's residue theorem to evaluate the integral of each of these functions around the circle $|z| = 3$ in the positive sense

   i. $\dfrac{e^{-z}}{z^2}$ iii. $z^2\exp\left(\dfrac{1}{z}\right)$

   ii. $\dfrac{e^{-z}}{(z-1)^2}$ iv. $\dfrac{z+1}{z^2-2z}$
8. Problems on Bilinear Transformation
   (a) Find the biliner transformation that maps the points z_1, z_2 and z_3 to w_1, w_2 and w_3 respectively as follows
      i. $z_1 = -1, z_2 = 0, z_3 = 1$ to $w_1 = -i, w_2 = 1, w_3 = i$.
      ii. $z_1 = 1, z_2 = 0, z_3 = -1$ to $w_1 = i, w_2 = \infty, w_3 = 1$.
   (b) Find the bilinear transformation that maps distinct points z_1, z_2, z_3 onto the points $w_1 = 0, w_2 = 1, w_3 = \infty$.
   (c) Find a bilinear map that transform the unit disk $|z| < 1$ onto the right half-plane, *i.e.*, $\text{Re } w > 0$.
   (d) Find a bilinear map that transform the unit disk $|z| < 1$ onto the upper half-plane, *i.e.*, $\text{Im } w > 0$.